Economic Model Technical Appendix
UNISECO deliverable 4.2

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1 Antecedents 1 - writing demand and supply in log derivative form

Inverse supply as a function of own price P and a production subsidy V per unit produced:

$$P = S^{-1}(Q_S) - V$$

Inverse demand as a function of own price P and a consumption tax C per unit produced:

$$P = D^{-1}(Q_D) - C$$

Take differentials:

$$dP = S'(Q_S)\, dQ_S - dV$$

$$dP = D'(Q_D)\, dQ_D - dC$$

Use definition of supply elasticity to substitute $S'(Q_S) = \frac{1}{\epsilon}\frac{P}{Q_S}$, $D'(Q_D) = \frac{1}{\eta}\frac{P}{Q_D}$ and rearrange:

$$\frac{dQ_S}{Q_S} = \epsilon\frac{dP}{P} + \epsilon\frac{dV}{P}$$

$$\frac{dQ_D}{Q_D} = \eta\frac{dP}{P} + \eta\frac{dC}{P}$$

Log differentiated prices quantities denoted by $EP = d\ln P = \frac{dP}{P}$ and $EQ = d\ln Q = \frac{dQ}{Q}$ respectively.

Log differentiated per-unit production and consumption subsidy denoted by $EV = d\ln(1+V) = \frac{dV}{P}$ and $EC = d\ln(1+C) = \frac{dC}{P}$ respectively, the change in the production subsidy or consumption tax rate as a percentage of price.

2 Antecedents 2 - log differentials for summed relationships

The economic model must somtimes make assumptions about quantity sums. For example, we need to assume that total production of a good equals domestic demand plus exports:

$$Q_S = Q_D + Q_X$$

Express as differential:

$$dQ_S = dQ_D + dQ_X$$

Divide both sides by Q_S:

$$\frac{dQ_S}{Q_S} = \frac{dQ_D}{Q_D}\frac{Q_D}{Q_S} + \frac{dQ_X}{Q_X}\frac{Q_X}{Q_S}$$

Express as shares:

$$EQ_S = (1 - \alpha_X)\, EQ_D + \alpha_X EQ_X$$

where $\alpha_X = \frac{Q_X}{Q_S}$. Note that $\frac{Q_D}{Q_S} = \frac{Q_S}{Q_S} - \frac{Q_X}{Q_S} = 1 - \alpha_X$.

3 Equilibrium displacement model of trade with 2 products

3.1 Basic Setup

2 products and 2 regions: EU product and RoW product

Each region produces its own product and consumes domestic and imported varieties, yielding 6 quantity flows.

Export supply is based on each region's domestic demand and supply elasticities

Superscript (EU or RoW) denotes the product, which differs by country of origin.

The EU can subsidize production of its own good by V per unit, and it can levy tariffs t^{RoW} on the imported good. The EU can also subsidise consumption by C per unit.

Price in exporting country or export quantity supplied denoted by subscript X, price in importing country or import quantity demanded denoted by subscript M. So the EU pays price P_X^{EU} for its own good, but pays price P_M^{RoW} for the good it imports from the RoW.

3.2 Import demand and export supply for EU good

Log-differentiated quantity supplied of the EU-produced good:

$$EQ_S^{EU} = \epsilon EP_X^{EU} + \epsilon EV \tag{1}$$

where EP_X^{EU} is the log-differentiated export price, ϵ is the own price elasticity of supply and EV is the change in the production subsidy as a share of the price.

Log-differentiated demand for own good in EU, :

$$EQ_D^{EU} = \eta EP_X^{EU} + \eta EC \tag{2}$$

where η is the own price elasticity of demand and EC is the change in the consumption tax as a share of the price.

(Log derivative) RoW import demand for EU good (M denotes imports):

$$EQ_M^{EU} = \eta_e EP_M^{EU} \tag{3}$$

where η_e is the elasticity of demand for imports. Note that we allow for the demand elasticity to differ between imports and the domestically-produced good.

We need an equation to relate changes in total EU production (EQ_S^{EU}) to changes in domestic demand EQ_D^{EU} and exports EQ_X^{EU}, in log-differentiated terms (see antecedents 2 for more detail):

$$EQ_S^{EU} = \left(1 - \alpha_X^{EU}\right) EQ_D^{EU} + \alpha_X^{EU} EQ_X^{EU} \tag{4}$$

where α_X^{EU} is the share of EU production that is exported.

Isolate EQ_X^{EU}:

$$EQ_X^{EU} = \frac{1}{\alpha_X^{EU}} EQ_S^{EU} - \frac{1 - \alpha_X^{EU}}{\alpha_X^{EU}} EQ_D^{EU} \tag{5}$$

Plug (1) and (2) into (5):

$$EQ_X^{EU} = \left(\frac{\epsilon - \eta}{\alpha_X^{EU}} + \eta\right) EP_X^{EU} + \frac{\epsilon}{\alpha_X^{EU}} EV - \eta \frac{1 - \alpha_X^{EU}}{\alpha_X^{EU}} EC \tag{6}$$

Equation (6) is the EU export supply equation. The quantity of EU exports is increasing with the price, increasing with productsion subsidies, and increasing with taxes on EU-produced goods. If EC is negative then the policy subsidizes consumption of EU-produced goods, and exports decrease.

3.3 Import demand and export supply for RoW good

RoW supply and demand: (assume same elasticities as EU)

RoW supply:

$$EQ_S^{RoW} = \epsilon EP_X^{RoW} \tag{7}$$

RoW demand for own good:

$$EQ_D^{RoW} = \eta EP_X^{RoW} \tag{8}$$

EU import demand for RoW good:

$$EQ_M^{RoW} = \eta_e EP_M^{RoW} \tag{9}$$

RoW export share α_X^{RoW}:

$$EQ_S^{RoW} = \left(1 - \alpha_X^{RoW}\right) EQ_D^{RoW} + \alpha_X^{RoW} EQ_X^{RoW}$$

Isolate EQ_X^{RoW}:

$$EQ_X^{RoW} = \frac{1}{\alpha_X^{RoW}} EQ_S^{RoW} - \frac{1 - \alpha_X^{RoW}}{\alpha_X^{RoW}} EQ_D^{RoW} \tag{10}$$

Plug (7) and (8) into (10):

$$EQ_X^{RoW} = \left(\frac{\epsilon - \eta}{\alpha_X^{RoW}} + \eta \right) EP_X^{RoW} \tag{11}$$

Equation (11) is the RoW export supply equation.

3.4 Market clearing conditions

We now need to define the market clearing conditions in log-differentiated format:

$$EQ_X^{EU} = EQ_M^{EU}$$
$$EQ_X^{RoW} = EQ_M^{RoW}$$

These market clearing conditions imply that we can write everything in terms of import quantities, substituting $EQ_X^{EU} = EQ_M^{EU}$ and $EQ_X^{RoW} = EQ_M^{RoW}$ in all the above expressions.

3.5 Arbitrage conditions

We also need to define the relationship between importer and exporter prices (arbitrage conditions) before log-differentiating:

$$P_M^{EU} = P_X^{EU}$$

$$P_M^{RoW} = P_X^{RoW} \left(1 + \tau^{RoW}\right)$$

where τ^{RoW} is ad valorem tariffs or trade costs.

Arbitrage conditions in log-differentiated terms:

$$EP_M^{EU} = EP_X^{EU} \tag{12}$$

$$EP_M^{RoW} = EP_X^{RoW} + t^{RoW} \tag{13}$$

where $t^{RoW} = d \ln \left(1 + \tau^{RoW}\right) = \frac{d\tau^{RoW}}{P}$.

3.6 Solving the model

Equations (3), (6), (9), (11), and (13) can be expressed in a matrix form:

$$\begin{bmatrix} 1 & 0 & -\delta & 0 & 0 \\ 0 & 1 & 0 & 0 & -\theta \\ 1 & 0 & -\eta_e & 0 & 0 \\ 0 & 1 & 0 & -\eta_e & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} EQ_M^{EU} \\ EQ_M^{RoW} \\ EP_M^{EU} \\ EP_M^{RoW} \\ EP_X^{RoW} \end{bmatrix} = \begin{bmatrix} \gamma \\ 0 \\ 0 \\ 0 \\ t^{RoW} \end{bmatrix}$$

where

$$\delta = \frac{\epsilon - \eta}{\alpha_X^{EU}} + \eta > 0$$

$$\theta = \frac{\epsilon - \eta}{\alpha_X^{RoW}} + \eta > 0$$

$$\gamma = \frac{\epsilon}{\alpha_X^{EU}} EV - \eta \frac{1 - \alpha_X^{EU}}{\alpha_X^{EU}} EC$$

$$EQ_M^{EU} = -\frac{\gamma \eta_e}{\delta - \eta_e}$$

$$EQ_M^{RoW} = \theta \frac{t^{RoW} \eta_e}{\theta - \eta_e}$$

$$EP^{EU} = -\frac{\gamma}{\delta - \eta_e}$$

$$EP_M^{RoW} = \frac{\theta t^{RoW}}{\theta - \eta_e}$$

$$EP_X^{RoW} = \frac{t^{RoW} \eta_e}{\theta - \eta_e}$$

Plug in our values for δ and θ:

$$EQ_M^{EU} = \frac{\eta_e \left(\eta \frac{1-\alpha_X^{EU}}{\alpha_X^{EU}} EC - \frac{\epsilon}{\alpha_X^{EU}} EV \right)}{\frac{\epsilon - \eta}{\alpha_X^{EU}} + \eta - \eta_e} \tag{14}$$

$$EQ_M^{RoW} = \left(\frac{\epsilon - \eta}{\alpha_X^{RoW}} + \eta \right) \frac{t^{RoW} \eta_e}{\frac{\epsilon - \eta}{\alpha_X^{RoW}} + \eta - \eta_e} \tag{15}$$

$$EP^{EU} = \frac{\eta \frac{1-\alpha_X^{EU}}{\alpha_X^{EU}} EC - \frac{\epsilon}{\alpha_X^{EU}} EV}{\frac{\epsilon - \eta}{\alpha_X^{EU}} + \eta - \eta_e} \tag{16}$$

$$EP_M^{RoW} = \frac{t^{RoW} \left(\frac{\epsilon - \eta}{\alpha_X^{RoW}} + \eta \right)}{\frac{\epsilon - \eta}{\alpha_X^{RoW}} + \eta - \eta_e} \tag{17}$$

$$EP_X^{RoW} = \frac{t^{RoW}\eta_e}{\frac{\epsilon-\eta}{\alpha_X^{RoW}} + \eta - \eta_e} \tag{18}$$

Without any constraints on domestic production and consumption in each region, equation (15) provides a unique solution for t^{RoW}, which we can see by rearranging:

$$t^{RoW} = EQ_M^{RoW} \frac{\frac{\epsilon-\eta}{\alpha_X^{RoW}} + \eta - \eta_e}{\eta_e\left(\frac{\epsilon-\eta}{\alpha_X^{RoW}} + \eta\right)}$$

We can also solve for some combination of EV and EC using (14). There are infinite combinations of EV and EC that match the EU export quantities, if we are not worried about matching the EU domestic production and consumption quantities. However, we need to match the BioBaM production and consumption quantities, which we discuss in the next section.

4 Constraining policies to match production and consumption quantitites from biophysical models

The biophysical models also stipulate the levels of production and consumption in each region for each storyline. We must thus constrain the solution from the economic model so that it matches production and consumption quantities as well.

Use the EU supply equation (1) and (14) to solve for EV and EC. This gives us two equations and two unknowns:

$$\frac{EQ_S^{EU} - \epsilon EV}{\epsilon} = EP^{EU}$$

$$EQ_M^{EU} = \eta_e EP^{EU}$$

Combine:

$$EV = \frac{EQ_S^{EU}}{\epsilon} - \frac{EQ_M^{EU}}{\eta_e}$$

This gives us the solution for EV.

We can then solve for EC using (2) and (14):

$$\frac{EQ_D^{EU} - \eta EC}{\eta} = EP^{EU}$$

$$EQ_M^{EU} = \eta_e EP^{EU}$$

Combine:

$$EC = \frac{EQ_D^{EU}}{\eta} - \frac{EQ_M^{EU}}{\eta_e}$$

5 Changes in economic surplus in the EU

Once we have solved for differences in policies and prices between the storylines, we can then calculate consumer surplus and producer surplus.

The change in EU consumer surplus for the domestically-produced good can be derived from the following formula:

$$\Delta CS_{dom}^{EU} = -P_0^{EU} Q_{D,0}^{EU} \cdot \left(EC + EP_X^{EU}\right)\left(1 + \frac{EQ_D^{EU}}{2}\right)$$

where P_0^{EU} and $Q_{D,0}^{EU}$ are the BAU price and quantity in the EU.

The change in EU consumer surplus for the imported good can be derived from the following formula:

$$\Delta CS_M^{EU} = -P^{EU} Q_M^{EU} \cdot \left(EP_M^{RoW}\right)\left(1 + \frac{EQ_M^{RoW}}{2}\right)$$

The change in EU producer surplus can be derived as following:

$$\Delta PS^{EU} = P^{EU} Q_S^{EU} \cdot \left(EV + EP_X^{EU}\right)\left(1 + \frac{EQ_S^{EU}}{2}\right)$$

The change in EU producer revenue can be derived as following:

$$\begin{aligned} \Delta \operatorname{Re} venue^{EU} &= P^{EU} Q_S^{EU} \cdot \left(EV + EP_X^{EU}\right)\left(1 + EQ_S^{EU}\right) \\ &\quad + P^{EU} Q_S^{EU} \cdot EQ_S^{EU} \end{aligned}$$